HYBRID CONTROL SYSTEMS

Karl Henrik Johansson
Royal Institute of Technology, Stockholm, Sweden

Keywords: Hybrid system; Real-time system; Dynamical system; Control theory; Embedded software; Hierarchical control; Computer-controlled system; Discrete-event systems; Hybrid automata; Discontinuous control; Switched system; Supervisory control; Automaton; Execution; Events; Verification; Finite state machine; Stability.

Contents

Summary

Most practical control systems involve both analogue and logic components. Hybrid system is a generic term for such systems, where the time-driven and event-driven dynamics are interacting. These systems arise naturally in many applications of automatic control, for example when a physical plant is controlled by a finite set of controls. The field of hybrid systems is a recent and very active research area, which evolves from the foundations of control theory and computer science. The developed framework is suitable for the design of complex embedded and networked control systems, which are often hard to analyze with traditional tools. Here a brief introduction to hybrid control is presented and the EOLSS articles under hybrid control systems are put into context.

1. Introduction

To find abstract models of technological phenomena is essential in many areas of engineering. For control system design, a good model is one that is not only comprehensive enough to capture the important system characteristics (dynamics, disturbances, measurement noise etc.), but is also simple enough at the same time to allow application of existing analysis and design methods. Traditional control has been devoted to continuous dynamical systems, where both plant and controller are modeled by differential equations. Despite the tremendous impact this approach has had on control design, it has several limitations because purely continuous time systems that can be well modeled by differential equations are far from all real control systems. Networked and embedded control systems are examples of emerging areas that have called for new modeling paradigms. High complexity and heterogeneous dynamics are common for them and most other application areas of hybrid systems. Hybrid systems provide a framework that often enables the designer to extract the desired properties of a system while ignoring irrelevant details. There also exist many applications that are of a truly hybrid nature, such as control systems with mode switches. Even though research projects in hybrid systems have covered a wide range of theoretical and practical topics, of which this chapter only discusses a few approaches, it is important to remember that the research area of hybrid systems is still young as most of the work was done mainly over the last ten years. Therefore, there does not yet exist a unified theory of hybrid systems (as for linear control systems and classes of nonlinear control systems), but the creation of such theory is very much an ongoing activity. See the bibliography at the end of the chapter for literature that discusses the evolution of hybrid systems and for major collections of papers on hybrid systems such as special issues of journals in automatic control.

1.1. Hybrid Systems

A *hybrid* system is in general a system composed of two unlike components. A *hybrid system* is a system with both analog and digital parts, in which time-driven (continuous command signal-driven) and event-driven dynamics interact. Such a system generates a mixture of continuous and discrete signals, which take values in a continuum (such as the real numbers $\mathbb{R}$) and a discrete set (such as $\{a,b,c,\ldots\}$), respectively. A *hybrid control system* is a control system, where the plant or the controller, individually, or in combination, can be modeled as a hybrid system.

A continuous-valued signal typically takes values in the set of real numbers (e.g., a temperature T taking values in the interval $[-20,100]$ degrees Celsius), while a discrete-valued signal takes value in a discrete (often finite) set (e.g., a thermostat valve V taking the values $\{\texttt{on},\texttt{off}\}$). The signals can either depend on time or be event-driven. A continuous-time signal is updated continuously through a differential equation (e.g., $\dot{T}(t) = -T(t)+1$) while a discrete-time signal is updated through a difference equation (e.g., $T(t+1) = T(t)+1$). An event-driven signal on the other hand is updated when an internal or external event happens (e.g., the switching on of the thermostat, $V' := \texttt{on}$, could be driven by the event that the temperature goes below 15 degrees, $T \leq 15$). Note that a discrete-time signal can be interpreted as an event-driven signal being updated when the event "sampling" takes place.

Let us develop a simple hybrid control system, which illustrates how continuous and discrete signals may interact. The example describes the problem of maintaining the temperature T of a room at some desired level (about 19 degrees Celsius, say). If the radiator is off, the temperature dynamics is given by

$$\dot{T} = -T + 15$$

and if it is on the temperature dynamics, is given by

$$\dot{T} = -T + 25.$$

It is convenient to represent a hybrid system by a graph. The hybrid system describing the heating of the room can be modeled as the graph shown in Figure 1. The two vertices of the graph represent the two discrete modes of the system: the radiator is either off or on.

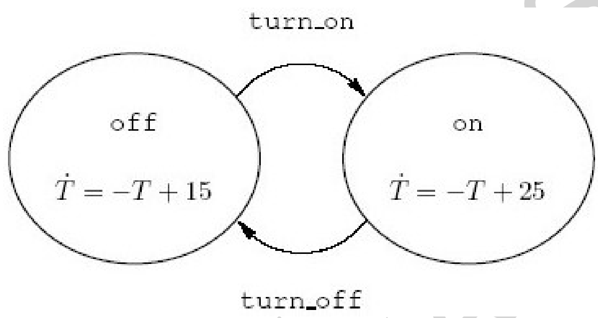

Figure 1. Hybrid control system modeling the heating of a room.

When the thermostat turns on the radiator, the hybrid system jumps from the off to the on state through the discrete transition `turn_on`. It jumps back again when the thermostat turns off the radiator. The temperature T, which is a continuous-time variable, is governed by one of the two differential equations depending on the current discrete state.

As long as the radiator is off, the temperature T will follow the dynamics specified in the left mode, i.e., T will tend to 15. When the event marked `turn_on` is triggered, the discrete state of the system jumps from the `off` to the `on` mode. The transition is represented by the upper edge of the graph. In the `on` mode, the temperature follows the dynamics given by the differential equation specified in the right vertex, i.e. T will tend to 25. When `turn_off` is triggered, the system returns to the `off` mode. The events `turn_on` and `turn_off` can be triggered by either external or internal signals. An external trigger can be when an operator manually turns the radiator `off`, or when a failure turns the switch `off`, or when a separate system (e.g., a heating system in another room) communicates the trigger. An internal trigger signal depends on the hybrid state of the system. As an example, suppose that a thermostat is used as a heat controller. An internal trigger can then be created by identifying the event `turn_on` with the logical condition $T \le 18$ and the event `turn_off` with the condition $T \ge 20$. For such a hybrid control system, the discrete transitions can take place only when these Boolean expressions are true. The state evolution of the hybrid control system is shown in Figure 2.

For the thermostat controlled system, it is easy to see that regardless of the initial state of the system (i.e., for all initial discrete state in {off, on} and initial continuous state $T(0) \in \mathbb{R}$), the temperature tends to oscillate between 18 and 20 degrees. For more complicated hybrid control systems such a conclusion is not obvious, but one needs theoretical and computational tools to analyze properties of hybrid control systems. These are further discussed in Section 3.

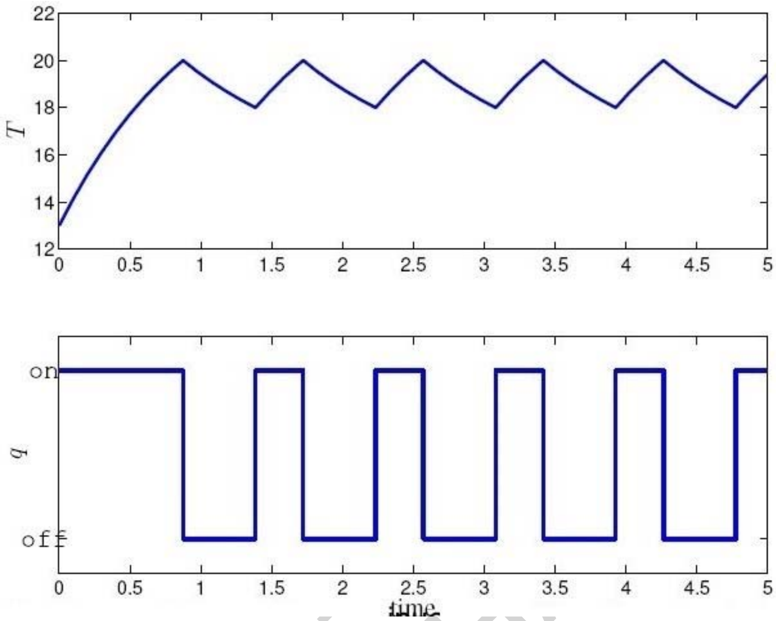

Figure 2: The evolution of the continuous and the discrete states of the hybrid control system in Figure 1.

Initially the temperature is equal to $T = 15$ and the radiator is in the mode $q = \texttt{on}$. The thermostat control mechanism switches the radiator off and on as T enables discrete jumps at $T = 18$ and 20, respectively.

1.2. Applications

It is important to capture the hybrid behavior of a control system when the time-driven and event-driven dynamics of the system interact in a way that influences the overall performance. This is the case in many applications, particularly when performance measures should be improved under sustained safety constraints. Such examples are ubiquitous in today's society. Hybrid system applications studied in some depth in the literature include air traffic management, chemical process control, embedded control, robotics, communication networks, and engine control. The hybrid nature is illustrated by the following examples:

- Air traffic management: A finite set of maneuvers, such as speed change, short cut, and detour, is used by the air traffic controller to obtain a conflict-free flight environment in which the aircrafts meet their dynamic and other constraints.

- Chemical process control: To produce a substance, an instruction sequence is designed, in which each instruction could involve one or more continuous control elements (e.g., keeping a reference temperature while a chemical reaction takes place).

- Embedded control: A micro-computer embedded in a physical device has an inherently discrete behavior (e.g., due to finite-precision computations and quantization of signals), but it interacts with a continuous environment through actuators and sensors.

- Robotics: A manipulator is accurately governed by continuous dynamics, but impacts and load shifting cause discrete-event changes.

- Communication networks: Large data flows are conveniently approximated as continuous variables (in order to reduce the model complexity), while over-full and empty buffers as well as some traffic control mechanisms induce discrete-event dynamics.

- Engine control: A four-stroke gasoline engine can be modeled using four discrete modes corresponding to the position of the pistons, while combustion and power train dynamics are continuous. In this case, the (complex) dynamics of the interaction between strokes is neglected, which can be useful in the analysis of certain phenomena.

The first four examples above illustrate systems which are hybrid either due to their plant dynamics or their controls, while the last two illustrate systems which naturally can be modeled as hybrid systems through time-scale separation or model reduction.

In classical control design, it is common to separate the event-driven and time-driven dynamics both in the analysis and in the design. For example, in a manufacturing process, one can model the processing of individual machines by their service times. In this way, the composed discrete-event system for the whole production line can be represented, for example, by a Petri net and analyzed using queuing theory. The control performance and quality assessment of each machine are modeled using continuous dynamics. The continuous feedback control is constrained by the service time specifications. If they are fulfilled, the higher-level discrete-event system is a suitable model for the overall evolution of the manufacturing system. The separation of asynchronous and synchronous controls will in many cases, however, lead to a too conservative design. In the manufacturing example, this could result in too large buffers and inefficient use of the machines. If instead all the dynamics and the interactions in the manufacturing process are captured within one single hybrid model, it is possible to optimize the overall behavior and thus achieve a high-performance design. Tools in hybrid control systems address this type of problems. In the development of these tools, it has been shown that similar type of improvement can be achieved in many different areas. A thorough discussion on the application of hybrid methods to air traffic management is given in *Case Study - Air Traffic Management Systems*. There it is shown not only that the hybrid system formalism allows to capture the dynamics of existing air traffic control systems, but also how new strategies enabling free flight can be derived.

-
-
-

Bibliography

Alur R., Courcoubetis C., Halbwachs N., Henzinger T., Ho P.H., Nicollin X., Olivero A., Sifakis J., Yovine S. (1995). The algorithmic analysis of hybrid systems. *Theoretical Computer Science* **138**, 3–34. [Defines hybrid automata and the corresponding verification problem.].

Alur R., Dill D. (1994). The theory of timed automata. *Theoretical Computer Science* **126**, 193–235. [Introduces timed automata for modeling real-time systems.].

Antsaklis P.J. (2000a). A brief introduction to the theory and applications of hybrid systems. *IEEE Proceedings* **88**(7), 879–887. [Introduction to hybrid systems and outline of special issue on hybrid control systems.].

— (2000b). Special issue on hybrid systems: Theory and applications. *IEEE Proceedings* **88**(7).

Antsaklis P.J., Nerode A. (1998). Special issue on hybrid control systems. *IEEE Transactions on Automatic Control* **43**(4).

Brockett R.W. (1993). Hybrid models for motion control systems. In H. Trentelman, J. Willems, eds., *Essays in Control: Perspectives in the Theory and Its Applications*, pp. 29–53, Birkhauser, Boston. [Introduces the use of hybrid systems for the modelling of motion control systems.].

Cassandras C.G. (1993). *Discrete Event Systems: Modeling and Performance Analysis*. Irwin Publ. [Discusses several models of interacting continuous and discrete-event dynamics, both deterministic and stochastic frameworks.].

Di Benedetto M.D. (2001). Special issue on hybrid systems in control. *International Journal of Robust and Nonlinear Control* **11**(5).

Evans R.J., Savkin A.V. (1999). Special issue on hybrid control systems. *Systems & Control Letters* **38**(3).

Hybrid Systems: Computation and Control (1998–2003). *Hybrid Systems: Computation and Control*. Lecture Notes in Computer Science, Springer-Verlag. [Proceedings of a major annual workshop on hybrid systems].

Lemmon M.D., He K.X., Markovsky I. (1999). Supervisory hybrid systems. *IEEE Control Systems Magazine* pp. 42–55. [Introductory text to hybrid control systems.].

Morse A.S., Pantelides C.C., Sastry S.S., Schumacher J.M. (1999). Special issue on hybrid systems. *Automatica* **35**(3).

van der Schaft A.J., Schumacher J.M. (1999). *An introduction to hybrid dynamical systems*. Lecture Notes in Control and Information Sciences 251, Springer-Verlag. [Introductory lecture notes in hybrid systems with an extensive discussion on complementarity systems and well-posedness.].

Biographical Sketch

Karl Henrik Johansson received the M.S. and Ph.D. degrees in electrical engineering, both from Lund University, Sweden, in 1992 and 1997, respectively. He held positions as Assistant Professor at Lund University (1997-1998) and as Visiting Research Fellow at the University of California, Berkeley (1998-2000). Currently, he is an Associate Professor in the Department of Signals, Sensors and Systems at the Royal Institute of Technology, Stockholm, Sweden. His research interests are in hybrid and switched systems, distributed embedded control, and applications in communication networks and automotive industry. Dr. Johansson received the Young Author Prize of the IFAC World Congress in 1996, the Peccei Award from IIASA, Austria, in 1993, and Young Researcher Award from Scania, Sweden, in 1996. He is an associate editor of Automatica.